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Optimization of Production Decisions Under Resource Constraints and Community Priorities

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ABSTRACT

The paper proposes an analytical solution of the problem of optimal allocation of resources in the game of two parties, based on the production function of Cobb-Douglas with constant returns under changing scale of production. The Edgeworth box is used to derive the main scientific findings of this research, namely the analytic representation of contract curve, community indifference curve, a production possibility frontier curve under Cobb-Douglas production function, and an analytical representation of the solution of maximization income in solving the resource allocation problem. The proposed technique is more suitable for practical application compared with prevailing theoretical resource allocation models based on Cobb-Douglas production function.

KEYWORDS

Cobb-Douglas, Community Indifference Curve, Contract Curve, Edgeworth Box, Production Possibility Frontier, Resource Allocation

1. INTRODUCTION

Production decisions are related to the resource allocation. This problem is obviously relevant from both theoretical and empirical perspectives as the resources are scarce in most cases (Ma, Zheng, 2019; Arif, 2021). Thus, it is important to relate the resource market and production technology to identify the production plans that are appealing for both producers and the society from the economic viewpoint.

The business environment is characterized with ever increasing competition in almost all aspects of economic activity (Tallman et al., 2018; Ozbekler & Ozturkoglu, 2020). Fluctuations in demand (Zhao & Zhu, 2018), disruptions in supply chains (Mahajan & Tomar, 2021), changes in consumer preferences (Sadílek, 2019) put additional pressure on producers, who have to adjust their business

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decisions accordingly. One of the ways to improve the overall performance of an economic agent is a more efficient resource allocation (Sibony et al., 2017).

Production technology can be modelled by exploiting the Cobb-Douglas production function (Kiselev & Orlov, 2010). Some research employs the latter approach in deriving the resource allocation models (Maialeh, 2019). The aim of this paper is to provide resource allocation model based on Cobb-Douglas production function.

The resource allocation model based on the Cobb-Douglas production function is relevant both in theoretical and practical terms. The proposed analytical form of the contract curve, community indifference curve, production possibility frontier and newly introduced concepts such as income contract curve and community income indifference curve constitute novel tools for modeling problems related to resource allocation. Analytical representation of the solution to the problem of maximizing income using a linearly homogeneous Cobb-Douglas production function allows for empirical analysis of a given system.

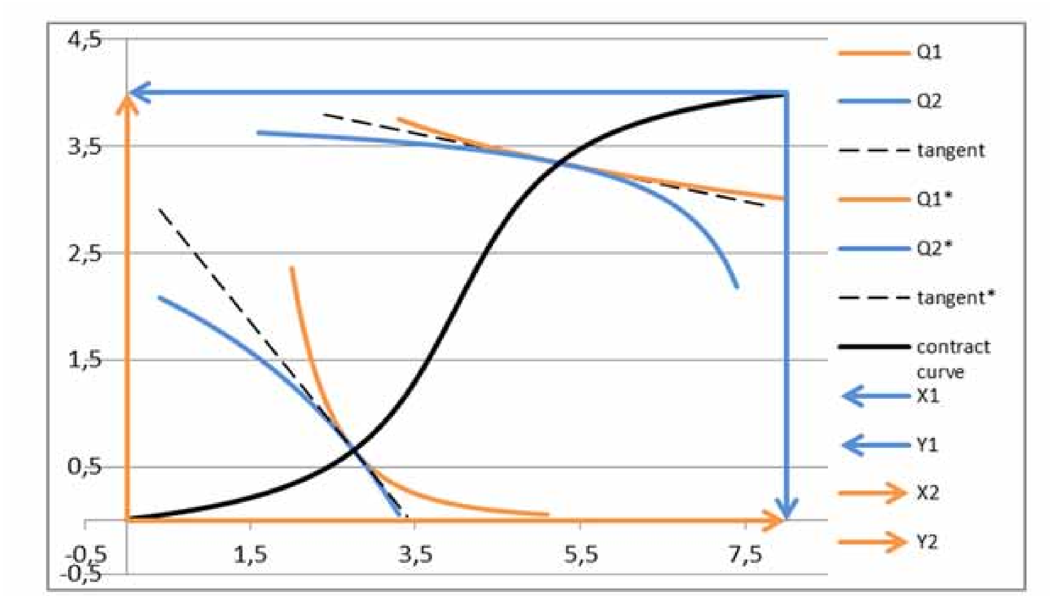
The paper is structured as follows: Section 2 presents the literature review on resource allocation problem, production possibility frontier and Edgeworth box. Section 3 provides details of the proposed model. Finally, some conclusions and research implications are offered in Section 4.

2. LITERATURE REVIEW

As one of the tools for determining the optimal allocation of resources in order to achieve various differing objectives in varying disciplines is the Edgeworth's box (Ricketts, 1986; Sprumont, 1995; Creedy, 2010; Suranovic, 2015; Doganova, 2015; Binmore, 2021). With the help of Edgeworth's box Choi (2009) seriously questions Rubinstein's bargaining theory. Anderton and Carter (2008) criticizes the classical Edgeworth box showing its limitations in depicting a vulnerable trade with goods subjected to appropriation. In order to increase its applicability and suitability for solving current economic problems, Gabaix (2014) propose to represent Edgeworth box as a two dimensional surface instead of one dimensional curve. His approach evolved to introduction of Edgeworth cube (Kunze & Schlatterer, 2018). Weiß (2021) significantly expanded application horizons of Edgeworth box based models by introducing them to time series data. Deda et al. (2013) use Edgeworth's box to simulate a competitive market equilibrium with the aim of deriving a production possibilities frontier. Tadenuma (2002) proves, what under condition of convexity in agents' preferences, the maximum elements aimed at efficiency-first appear in Edgeworth box. Purely market driven resource allocation and optimization incentives dominate Gode et al. (2004) double auction game based on Edgeworth's box. Sairamesh et al. (1995) use Edgeworth's box in a search for Pareto optimum in determining the service quality. Viane et al. (2002) choose Edgeworth box as a tool solving insurer-client bargaining game. Edgeworth box is a primary tool for finding an optimum equilibrium for the risky assets in capital markets (Shalit &Yitzhaki, 2009). Various bilateral cooperation agreements arousing in close border cooperation and its Pareto optimum's are investigated by Zaman (2011) employing an Edgeworth's box approach. The interdisciplinary (physics and economics) approach in explaining aggregate trade of goods functions based on Boltzmann-type equations supplemented by Edgeworth box was introduced by Toscani et al. (2013). Although not free of limitations, Bimonte and Punzo (2016) proposed host-guest interaction model based on Edgeworth box allows integration it into various related disciplines, not purely economics. Toda & Walsh (2017) address to Edgeworth's box when investigating multiple equilibria of Bernoulli utilities in perfect competition condition. Araujo et al. (2018) use Edgeworth box in finding determinants for Arrow–Debreu equilibria. Glötzl et al. (2019) modify the Edgeworth box introducing a gradient climbing. Such an optimization allows to better investigate behavioural decisions which are based not on a perfect, but imperfect rationality and bias. Zweifel (2019) employ Edgeworth box with a purpose to investigate the possible success of public health system reforms. The citizen-government interaction under strain conditions (COVID-19 pandemic) was investigated by Zweifel (2020) using Edgeworth box as a primary research instrument.

These and other studies use the theoretical form of the Edgeworth box and the contract curve (Fig. 1). The resulting analytical expression of the CC allows to refine its graphical form.

Figure 1. Edgeworth box for Q_1 and Q_2 (theoretical).



3. MODEL BUILDING

This chapter considers the problem of optimal resource allocation for linearly homogeneous production functions of the Cobb-Douglas type. The work defines for two production functions with analytical view - contract curve (CC); - community indifference curve (CIC); - production possibility frontier (PPF); - solving the problem of optimal resource allocation. The concept is introduced and the analytical form is defined by community income indifference curve (CIIC) and income contract curve (ICC). The abbreviations and notations used are presented in Table 1.

Suppose there are two types of resources with the maximum volume *L* and *K*. Thus, there are two linearly homogeneous production functions of the Cobb-Douglas:

$$Q_1\left(X_1,\ Y_1\right) = \beta_1 X_1^{\alpha_1} Y_1^{1-\alpha_1}, Q_2\left(X_2,\ Y_2\right) = \beta_2 X_2^{\alpha_2} Y_2^{1-\alpha_2} \tag{1}$$

In this case, the constraints $X_1 + X_2 \leq L$, $Y_1 + Y_2 \leq K$ are fulfilled. These two products are sold at prices p_1 and p_2, respectively. It is necessary to distribute the limited resources L and K in such a way that the total income $I\left(Q_1, Q_2\right)$ from the sale of products of volume Q_1 and Q_2 is maximum. Formally, the problem can be written as follows:

Table 1. Abbreviations and notations used

B	Contract curve
ICC	Income contract curve
CIC	Community indifference curve
CIIC	Community income indifference curve
PPF	Production possibility frontier
L	Maximum quantity of the first resource
K	Maximum quantity of the second resource
Q_i	Production amount of product i, $i = 1,2$
X_i	Quantity of the first resource used for production of product i, $i = 1,2$
Y_i	Quantity of the second resource used for production of product i, $i = 1,2$
$\alpha_i,\ \beta_i$	Parameters of the Cobb-Douglas function $Q_i\left(X_i, Y_i\right) = \beta_i X_i^{\alpha_i} Y_i^{1-\alpha_i}$, $i = 1,2$
p_i	Price of product i, $i = 1,2$
I_i	Sales of product i, $i = 1,2$
I	Total sales revenue

$$\begin{cases} I\left(Q_1, Q_2\right) = I_1\left(Q_1\right) + I_2\left(Q_2\right) = p_1 Q_1 + p_2 Q_2 \to max \\ Q_1\left(X_1,\ Y_1\right) = \beta_1 X_1^{\alpha_1} Y_1^{1-\alpha_1},\ Q_2\left(X_2,\ Y_2\right) = \beta_2 X_2^{\alpha_2} Y_2^{1-\alpha_2} \\ 0 \le X_1 + X_2 \le L,\ 0 \le Y_1 + Y_2 \le K \end{cases} \tag{2}$$

The contract curve consists of points at which the lines of functions (1) touch each other. At these points the angles of the tangents to functions (1) are equal. Therefore, the derivatives to functions (1) are equal at these points. Now the equations of functions can be written down (1) in the conditions of constructing the Edgeworth box (conditions of constraints on resources used a depicted in (2)).

$$Q_1\left(X_2,\ Y_2\right) = \beta_1\left(L - X_2\right)^{\alpha_1}\left(K - Y_2\right)^{1-\alpha_1}, Q_2\left(X_2,\ Y_2\right) = \beta_2 X_2^{\alpha_2} Y_2^{1-\alpha_2} \tag{3}$$

Thus, it is possible to find and equate the derivatives of implicitly defined functions (3):

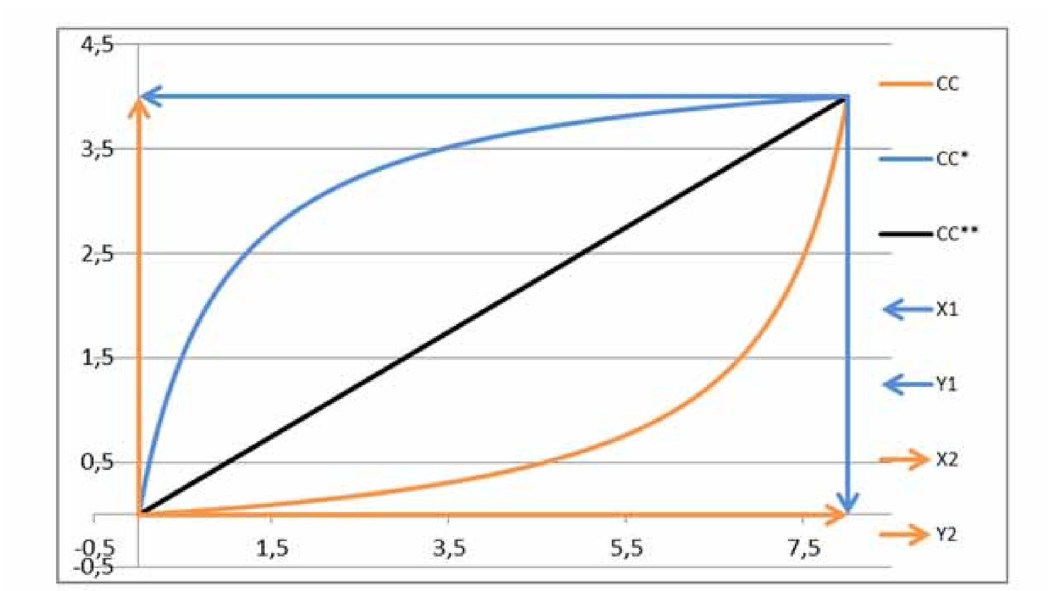
$$-\frac{\alpha_1}{1-\alpha_1}\frac{K - Y_2}{L - X_2} = -\frac{\alpha_2}{1-\alpha_2}\frac{Y_2}{X_2} \tag{4}$$

Let $\gamma_{12} = \frac{\left(1-\alpha_1\right)\alpha_2}{\alpha_1\left(1-\alpha_2\right)}$, then from (4) the analytical form of the contract curve can be obtained:

$$Y_2 = \frac{KX_2}{\gamma_{12}L + (1-\gamma_{12})X_2} \tag{5}$$

It can be stated that the graph of function (5) within the boundaries of the Edgeworth box will be a part of the branch of the hyperbola (Fig. 2).

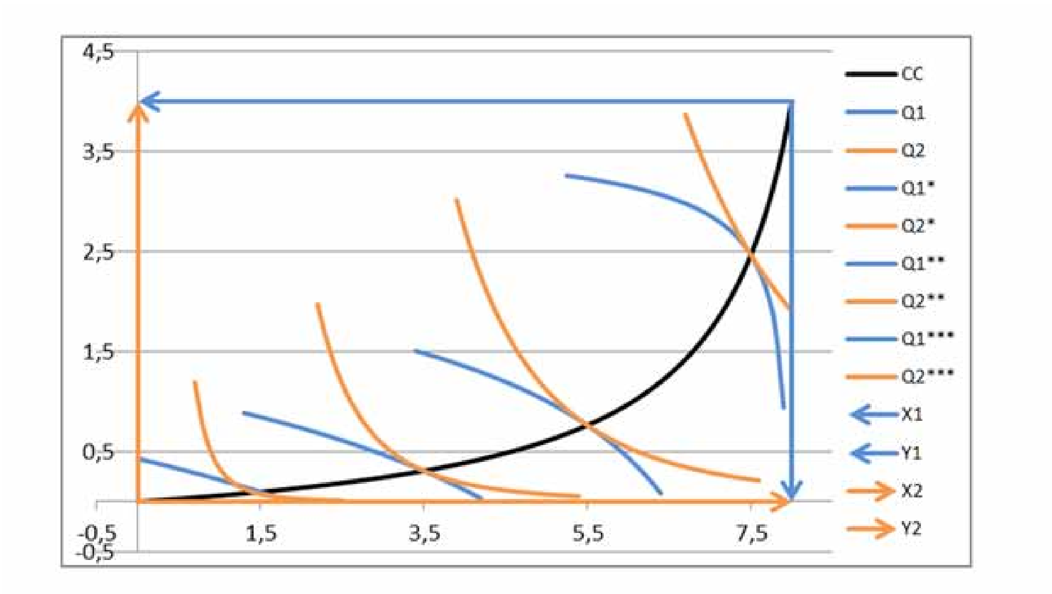
Figure 2. Possible options for contract curves using formula (5)



With $\alpha_1 = \alpha_2$ ($\gamma_{12} = 1$), the contract curve will be the straight line $Y_2 = \frac{K}{L}X_2$ (CC **, Fig. 2). The type of the CC * contract curve (Fig. 2) can be obtained from the CC curve by renumbering the production functions. For $\alpha_1 < \alpha_2$ ($\gamma_{12} > 1$), the contract curve will have the CC form. For $\alpha_1 > \alpha_2$ ($\gamma_{12} < 1$), the contract curve will have the form CC*. Therefore, it can be argued that the contract curve in the general case will have the form of the CC shown in Fig. 3 Thus, it is assumed $\alpha_1 < \alpha_2$ ($\gamma_{12} > 1$).

Employing analytical approach the fixing of the income for the second type of product $I_2\left(Q_2\right)$ is possible. Then the optimal distribution of resources will be obtained by maximizing income for the first type of product $I_1\left(Q_1\right)$. Thus, it is necessary to solve the problem:

Figure 3. General view of the CC contract curve by formulas (3) and (5)



$$\begin{cases} I_1\left(Q_1\right) = p_1 Q_1 \rightarrow max \\ Q_1\left(X_1,\ Y_1\right) = \beta_1 X_1^{\alpha_1} Y_1^{1-\alpha_1}, \\ 0 \le X_1 + X_2 \le L,\ 0 \le Y_1 + \left(\dfrac{I_2\left(Q_2\right)}{p_2 \beta_2 X_2^{\alpha_2}}\right)^{\frac{1}{1-\alpha_2}} \le K \end{cases} \tag{6}$$

This problem is equivalent to the following problem:

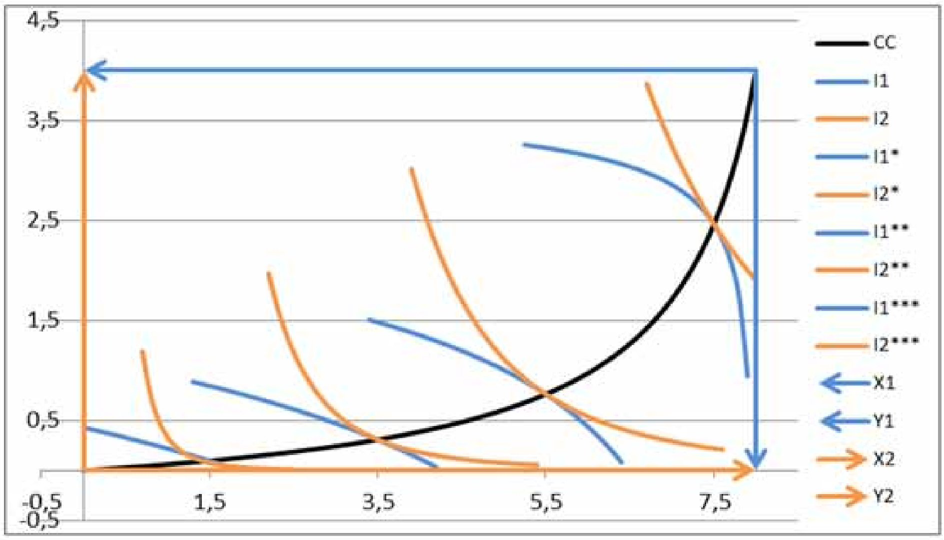
$$I_1\left(X_2\right) = p_1 \beta_1 \left(L - X_2\right)^{\alpha_1} \left(K - \left(\frac{I_2\left(Q_2\right)}{p_2 \beta_2 X_2^{\alpha_2}}\right)^{\frac{1}{1-\alpha_2}}\right)^{1-\alpha_1} \rightarrow max \tag{7}$$

The solution to problem (7) can be found from the solution of the next equation:

$$\frac{dI_1\left(X_2\right)}{dX_2} = 0 \tag{8}$$

Defining the type of community indifference curve (CIC) or Scitovsky indifference curve (SIC), it is supposed that a certain state on the contract curve is realized under the given constraints

Figure 4. Edgeworth box for I_1 and I_2



on the resources L and K. In this case, the incomes are equal to $\overline{I_1}$ and $\overline{I_2}$. Let us determine the possible combinations (L, K) at which the incomes remain the same (CIC).

Graphical Approach (Scitovsky, 1942) says that to get a new combination of resources (L, K) for which the income will remain the same, it is necessary to move the graph $\overline{I_1}$ to the left (right) by ΔX and up (down) by ΔY in the Edgeworth box until a new touch with the graph and $\overline{I_2}$. The point (L, K) (the origin of the coordinate system for $\overline{I_1}$) is moved by the same distances. Since the graph is transferred at the same time as the coordinate system, the values of $\overline{TR_1}$ will not change. We get a new Edgeworth box, where the new value of the origin of the coordinate system for $\overline{I_1}$ $\left(L^*, K^*\right)$ will belong to CIC (Fig. 5).

Using an analytical approach, we need to make a series of steps

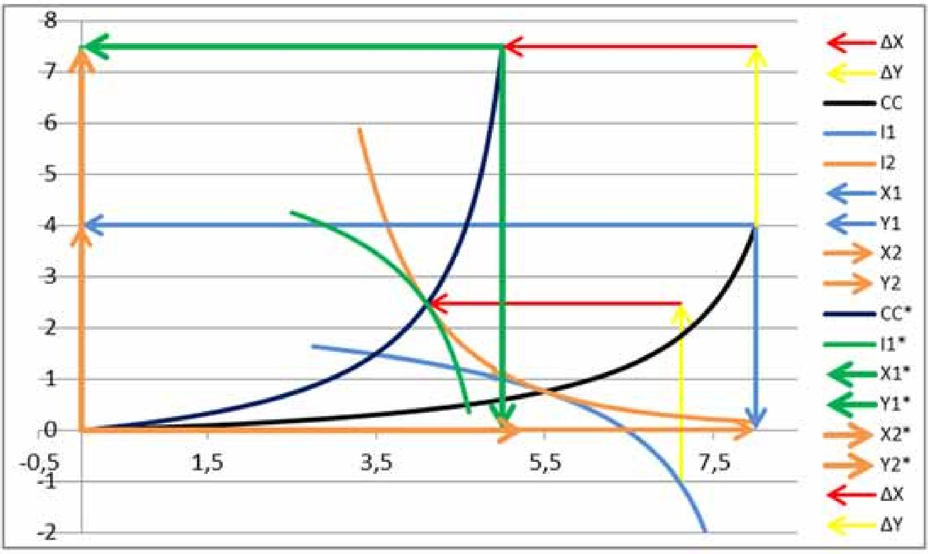
Using formula (2), we obtain the following equations

$$\overline{I_1} = p_1\beta_1\left(\frac{K - Y_2}{L - X_2}\right)^{1-\alpha_1}\left(L - X_2\right) \tag{9}$$

$$\overline{I_2} = p_2\beta_2\left(\frac{Y_2}{X_2}\right)^{1-\alpha_2} X_2 \tag{10}$$

According to formulas (4) and (5) we get

Figure 5. Graphical construction of the CIC using formulas (2) and (5).



$$\frac{K-Y_2}{L-X_2}=\gamma_{12}\frac{Y_2}{X_2} \tag{11}$$

Then formula (10) can be transformed:

$$\frac{Y_2}{X_2}=\left(\frac{\overline{I}_2}{p_2\beta_2}\right)^{\frac{1}{1-\alpha_2}}X_2^{-\frac{1}{1-\alpha_2}} \tag{12}$$

Thus, using formulas (9), (11) and (12) the following formula can be obtained

$$\overline{I}_1=p_1\beta_1\gamma_{12}^{1-\alpha_1}\left(\left(\frac{\overline{I}_2}{p_2\beta_2}\right)^{\frac{1}{1-\alpha_2}}X_2^{-\frac{1}{1-\alpha_2}}\right)^{1-\alpha_1}\left(L-X_2\right) \tag{13}$$

From (13) an expression for L can be obtained:

$$L = \frac{\left(\frac{\overline{I}_1}{p_1\beta_1}\right)}{\gamma_{12}^{1-\alpha_1}\left(\frac{\overline{I}_2}{p_2\beta_2}\right)^{\frac{1-\alpha_1}{1-\alpha_2}}} X_2^{\frac{1-\alpha_1}{1-\alpha_2}} + X_2 \tag{14}$$

Then, using formula (2) the following can be obtained:

$$\overline{I}_1 = p_1\beta_1\left(\frac{K-Y_2}{L-X_2}\right)^{-\alpha_1}\left(K-Y_2\right) \tag{15}$$

And consequently by using (11) and (12) we get

$$\overline{I}_1 = p_1\beta_1\gamma_{12}^{-\alpha_1}\left(\frac{\overline{I}_2}{p_2\beta_2}\right)^{-\frac{\alpha_1}{1-\alpha_2}} X_2^{\frac{\alpha_1}{1-\alpha_2}}\left(K-\left(\frac{\overline{I}_2}{p_2\beta_2}\right)^{\frac{1}{1-\alpha_2}} X_2^{-\frac{\alpha_2}{1-\alpha_2}}\right) \tag{16}$$

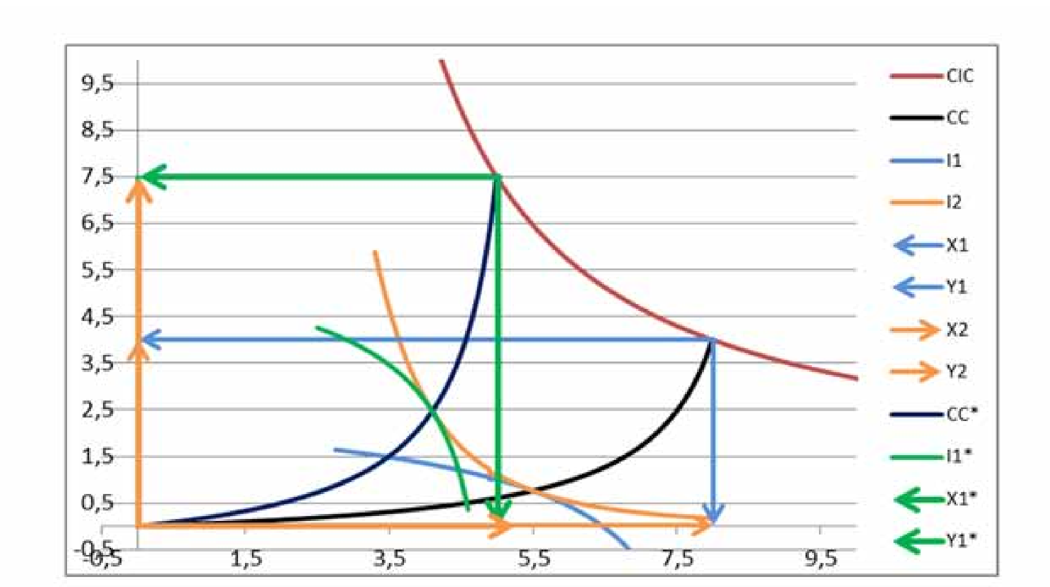
Thus from (16) an expression for K is obtained:

$$K = \gamma_{12}^{\alpha_1}\left(\frac{\overline{I}_1}{p_1\beta_1}\right)\left(\frac{\overline{I}_2}{p_2\beta_2}\right)^{\frac{\alpha_1}{1-\alpha_2}} X_2^{-\frac{\alpha_1}{1-\alpha_2}} + \left(\frac{\overline{I}_2}{p_2\beta_2}\right)^{\frac{1}{1-\alpha_2}} X_2^{-\frac{\alpha_2}{1-\alpha_2}} \tag{17}$$

By this approach, a parametrically specified function $K = f(L)$ is obtained, the graph of which will be CIC (Fig. 6):

$$\begin{cases} L = \dfrac{\left(\dfrac{\overline{I}_1}{p_1\beta_1}\right)}{\gamma_{12}^{1-\alpha_1}\left(\dfrac{\overline{I}_2}{p_2\beta_2}\right)^{\frac{1-\alpha_1}{1-\alpha_2}}} X_2^{\frac{1-\alpha_1}{1-\alpha_2}} + X_2 \\ K = \gamma_{12}^{\alpha_1}\left(\dfrac{\overline{I}_1}{p_1\beta_1}\right)\left(\dfrac{\overline{I}_2}{p_2\beta_2}\right)^{\frac{\alpha_1}{1-\alpha_2}} X_2^{-\frac{\alpha_1}{1-\alpha_2}} + \left(\dfrac{\overline{I}_2}{p_2\beta_2}\right)^{\frac{1}{1-\alpha_2}} X_2^{-\frac{\alpha_2}{1-\alpha_2}} \\ X_2 \in R^+ \end{cases} \tag{18}$$

Figure 6. Analytical construction of CIC using formulas (2), (5), and (18).



Having defined such parameters, it can be proceeded to the creation of a production possibility frontier (PPF). For this, we initially transform formulas (3) and (5)

Using formula (5) can be obtained:

$$K - Y_2 = \frac{\gamma_{12} K \left(L - X_2 \right)}{\gamma_{12} L + (1 - \gamma_{12}) X_2} \tag{19}$$

And formula (3):

$$I_1 \left(X_2,\ Y_2 \right) = p_1 \beta_1 \left(L - X_2 \right)^{\alpha_1} \left(K - Y_2 \right)^{1-\alpha_1}, I_2 \left(X_2,\ Y_2 \right) = p_2 \beta_2 X_2^{\alpha_2} Y_2^{1-\alpha_2} \tag{20}$$

Substituting expressions (5) and (19) into (20), one can obtain the parametric specification of PPF:

$$\begin{cases} I_1 \left(X_2 \right) = \dfrac{p_1 \beta_1 \gamma_{12}^{1-\alpha_1} K^{1-\alpha_1} \left(L - X_2 \right)}{\left(\gamma_{12} L + (1 - \gamma_{12}) X_2 \right)^{1-\alpha_1}} \\ I_2 \left(X_2 \right) = \dfrac{p_2 \beta_2 K^{1-\alpha_2} X_2}{\left(\gamma_{12} L + (1 - \gamma_{12}) X_2 \right)^{1-\alpha_2}} \\ \quad 0 \le X_2 \le L \end{cases} \tag{21}$$

In the case $\alpha_1 = \alpha_2 = \alpha$ ($\gamma_{12} = 1$), for which CC is a straight line $Y_2 = \dfrac{K}{L} X_2$, the PPF equation can be set explicitly:

$$I_2 \left(I_1 \right) = p_2 \beta_2 L^{\alpha} K^{1-\alpha} - \frac{p_2 \beta_2}{p_1 \beta_1} I_1 \tag{22}$$

The graph of expression (22) is a part of the straight line (Fig. 7). Graph (21) is a quasiconvex curve (Fig. 8).

Having determined all the necessary parameters, we proceed to solving the problem of optimal resource allocation

In the case $\alpha_1 = \alpha_2 = \alpha$ ($\gamma_{12} = 1$) for which the SS is a direct $Y_2 = \dfrac{K}{L} X_2$ problem (2) turns into a new problem:

$$\begin{cases} I = I_2 \left(L, K \right) + \left(1 - \dfrac{p_2 \beta_2}{p_1 \beta_1} \right) I_1 \left(X_2 \right) \to max \\ I_1 \left(X_2 \right) = p_1 \beta_1 \left(\dfrac{K}{L} \right)^{1-\alpha} \left(L - X_2 \right),\ I_2 \left(L,\ K \right) = p_2 \beta_2 L^{\alpha} K^{1-\alpha} \\ \quad 0 \le X_2 \le L \end{cases} \tag{23}$$

For $p_2\beta_2 < p_1\beta_1$, the objective function in (23) grows with an increase in $I_1\left(X_2\right)$ and reaches its maximum at $X_2 = 0$. Thus, solution (23) will be (PPF *, Fig. 7):

$$\left(X_1^0, Y_1^0\right) = \left(L, K\right),\ \left(X_2^0, Y_2^0\right) = \left(0,0\right),\ I_1^0 = p_1\beta_1 L^\alpha K^{1-\alpha},\ I_2^0 = 0,\ I^0 = p_1\beta_1 L^\alpha K^{1-\alpha}$$

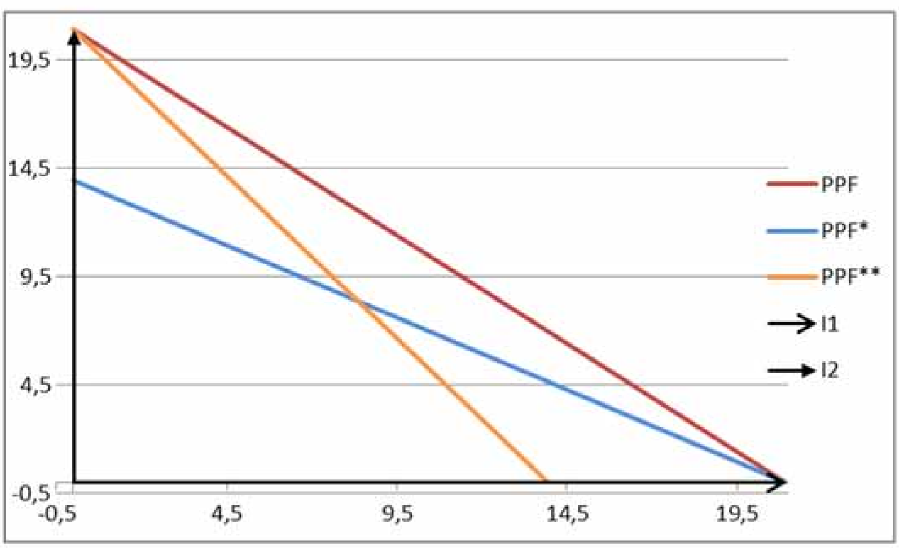
For $p_2\beta_2 > p_1\beta_1$, the objective function in (23) decreases with an increase in $I_1\left(X_2\right)$, therefore, reaches its maximum at $X_2 = L$. Thus, solution (23) will be (PPF **, Fig. 7):

$$\left(X_1^0, Y_1^0\right) = \left(0,0\right),\ \left(X_2^0, Y_2^0\right) = \left(L, K\right), I_1^0 = 0,\ I_2^0 = p_2\beta_2 L^\alpha K^{1-\alpha},\ I^0 = p_2\beta_2 L^\alpha K^{1-\alpha}.$$

In the degenerate case, for $p_2\beta_2 = p_1\beta_1 = p\beta$, the objective function in (23) is constant; therefore, it reaches its maximum $p_1\beta_1 L^\alpha K^{1-\alpha} = p_2\beta_2 L^\alpha K^{1-\alpha} = p\beta L^\alpha K^{1-\alpha}$ for any collection $\left(X_2, Y_2\right)$ from CC. Thus, solution (23) will be (PPF, Fig. 7):

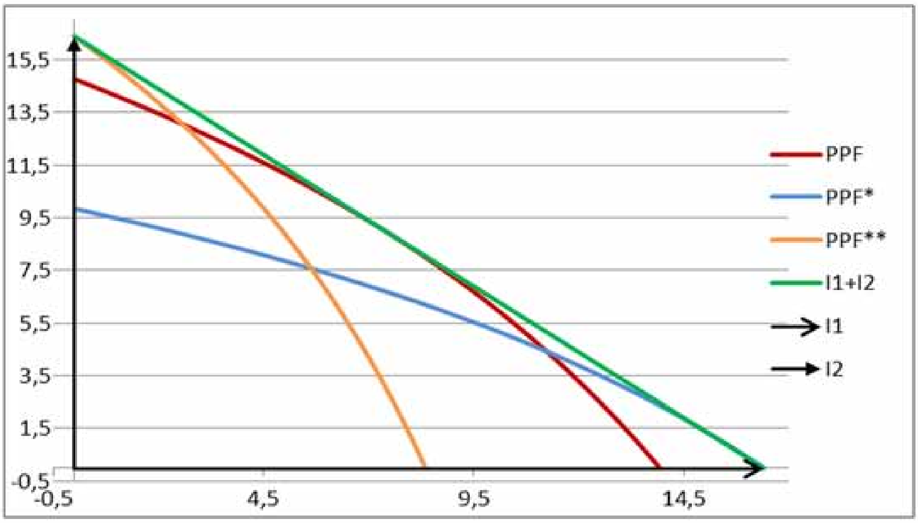
$$\left(X_1^0, Y_1^0\right) = \left(L - X_2, \frac{K}{L}\left(L - X_2\right)\right),\ \left(X_2^0, Y_2^0\right) = \left(X_2, \frac{K}{L} X_2\right),\ I_1^0 = p_1\beta_1\left(\frac{K}{L}\right)^{1-\alpha}\left(L - X_2\right),\ I_2^0 = p_2\beta_2\left(\frac{K}{L}\right)^{1-\alpha} X_2,\ I^0 = p\beta L^\alpha K^{1-\alpha}.$$

Figure 7. Solution of the optimal distribution problem with $\alpha_1 = \alpha_2$ for different PPF options by using formula (22)



For $\alpha_1 < \alpha_2$ ($\gamma_{12} > 1$) for which CC is a hyperbola (5), there are three possible cases for solving problem (2): two angular solutions (PPF *, PPF **, Fig. 8) and one internal solution (PPF, Fig. 8). Now it is necessary to define the conditions for these cases and the optimal solutions.

Figure 8. Solution of the problem of optimal resource allocation with $\alpha_1 < \alpha_2$ for different PPF options using formula (21)



Thus, using formula (5), the following can be obtained:

$$K - Y_2 = \frac{\gamma_{12} K \left(L - X_2\right)}{\gamma_{12} L + (1 - \gamma_{12}) X_2} \tag{24}$$

Substituting expressions (5) and (24) into the objective function (2), the new problem is set:

$$\begin{cases} I\left(X_2\right) = p_1 \beta_1 \gamma_{12}^{1-\alpha_1} K^{1-\alpha_1} \dfrac{\left(L - X_2\right)}{\left(\gamma_{12} L + (1 - \gamma_{12}) X_2\right)^{1-\alpha_1}} + \\ \quad p_2 \beta_2 K^{1-\alpha_2} \dfrac{X_2}{\left(\gamma_{12} L + (1 - \gamma_{12}) X_2\right)^{1-\alpha_2}} \to max \\ \qquad 0 \le X_2 \le L \end{cases} \tag{25}$$

In order to solve problem, formula 25 needs to be transformed:

$$\frac{dI\left(X_2\right)}{dX_2} = \frac{\left(1 - \alpha_1\right) L + \left(\alpha_1 - \alpha_2\right) X_2}{\left(1 - \alpha_2\right)} \left(- \frac{p_1 \beta_1 \gamma_{12}^{1-\alpha_1} K^{1-\alpha_1}}{\left(\gamma_{12} L + (1 - \gamma_{12}) X_2\right)^{2-\alpha_1}} + \frac{p_2 \beta_2 K^{1-\alpha_2} \alpha_2}{\alpha_1 \left(\gamma_{12} L + (1 - \gamma_{12}) X_2\right)^{2-\alpha_2}} \right) = 0 \tag{26}$$

Since $\left(1 - \alpha_1\right) L + \left(\alpha_1 - \alpha_2\right) X_2 > 0$ under conditions (25), from (26) is obtained:

$$\gamma_{12}L + (1-\gamma_{12})X_2 = \left(\frac{p_1\beta_1\alpha_1}{p_2\beta_2\alpha_2}\right)^{\frac{1}{\alpha_2-\alpha_1}} \gamma_{12}^{\frac{1-\alpha_1}{\alpha_2-\alpha_1}} K \tag{27}$$

Let's denote $\delta_{12} = \left(\frac{p_1\beta_1\alpha_1}{p_2\beta_2\alpha_2}\gamma_{12}^{1-\alpha_1}\right)^{\frac{1}{\alpha_2-\alpha_1}}$, then (27) will have the form:

$$\gamma_{12}L + (1-\gamma_{12})X_2 = \delta_{12}K \tag{28}$$

From (28) the optimal value X_2^0 и X_1^0 can be found:

$$X_2^0 = \frac{1}{1-\gamma_{12}}\left(\delta_{12}K - \gamma_{12}L\right) \tag{29}$$

$$X_1^0 = L - X_2^0 = \frac{1}{1-\gamma_{12}}\left(L - \delta_{12}K\right) \tag{30}$$

From (5) and (24), using expressions (28), (29), (30), one can find the optimal value Y_2^0 и Y_1^0:

$$Y_2^0 = \frac{1}{\left(1-\gamma_{12}\right)\delta_{12}}\left(\delta_{12}K - \gamma_{12}L\right) \tag{31}$$

$$Y_1^0 = K - Y_2^0 = \frac{\gamma_{12}}{\left(1-\gamma_{12}\right)\delta_{12}}\left(L - \delta_{12}K\right) \tag{32}$$

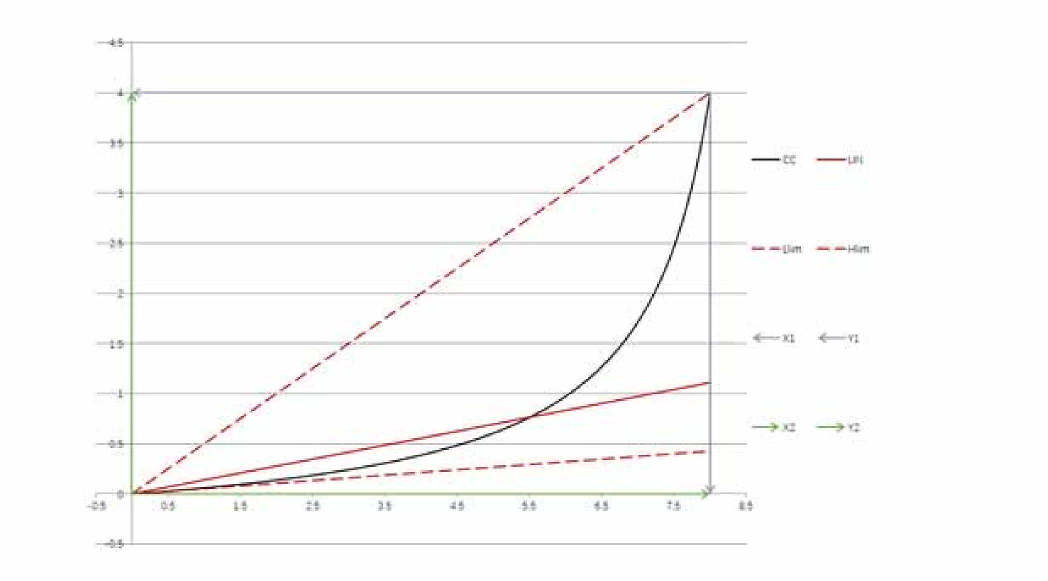
Substituting (28) into (5), one can find that solutions to problem (2) are at the intersection of the hyperbola (CC) and the straight line (LIN) (33) (Fig. 9):

$$Y_2 = \frac{X_2}{\delta_{12}} \tag{33}$$

The internal solution to problem (2) will take place when a straight line (LIN) is found within the range from the lower bound (Llim), given by the tangent to the SS at (0,0), to the upper bound (Hlim), given by the diagonal of the Edgeworth box (Fig. 9). From the constraint of problem (25), we obtain the conditions for the existence of an internal solution:

$$1 < \delta_{12}\frac{K}{L} < \gamma_{12} \tag{34}$$

Figure 9. Graphical definition of the optimal solution to problem (2) on the CC using formulas (5), (33), (34)



Under the condition $\delta_{12}\dfrac{K}{L} \geq \gamma_{12}$, the objective function in (25) reaches its maximum at $X_2 = 0$. Thus, solution (25) will be (PPF *, Fig. 8):

$$\left(X_1^0, Y_1^0\right) = \left(L, K\right),\ \left(X_2^0, Y_2^0\right) = \left(0, 0\right),\ \ I_1^0 = p_1\beta_1 L^{\alpha_1} K^{1-\alpha_1},\ \ I_2^0 = 0,\ \ I^0 = p_1\beta_1 L^{\alpha_1} K^{1-\alpha_1}.$$

When the condition $\delta_{12}\dfrac{K}{L} \leq 1$ is satisfied, the objective function in (25) reaches its maximum at $X_2 = L$. Thus, solution (25) will be (PPF **, Fig. 8):

$$\left(X_1^0, Y_1^0\right) = \left(0, 0\right),\ \left(X_2^0, Y_2^0\right) = \left(L, K\right),\ I_1^0 = 0,\ I_2^0 = p_2\beta_2 L^{\alpha_2} K^{1-\alpha_2},\ I^0 = p_2\beta_2 L^{\alpha_2} K^{1-\alpha_2}.$$

If the condition $1 < \delta_{12}\dfrac{K}{L} < \gamma_{12}$ is fulfilled, solution (25) will be (PPF, Fig. 8): $\left(X_1^0, Y_1^0\right)$ - formulas (30), (32); $\left(X_2^0, Y_2^0\right)$ - formulas (29), (31);

$$I_1^0 = \frac{p_1\beta_1\gamma_{12}^{1-\alpha_1}}{\left(1-\gamma_{12}\right)\left(\delta_{12}\right)^{1-\alpha_1}}\left(L - \delta_{12}K\right) \tag{35}$$

$$I_2^0 = \frac{p_2\beta_2}{\left(1-\gamma_{12}\right)\left(\delta_{12}\right)^{1-\alpha_2}}\left(\delta_{12}K - \gamma_{12}L\right) \tag{36}$$

$$I^0 = \frac{p_2\beta_2}{\left(\delta_{12}\right)^{1-\alpha_2}}\left(\alpha_2 L + \left(1-\alpha_2\right)\delta_{12}K\right) \tag{37}$$

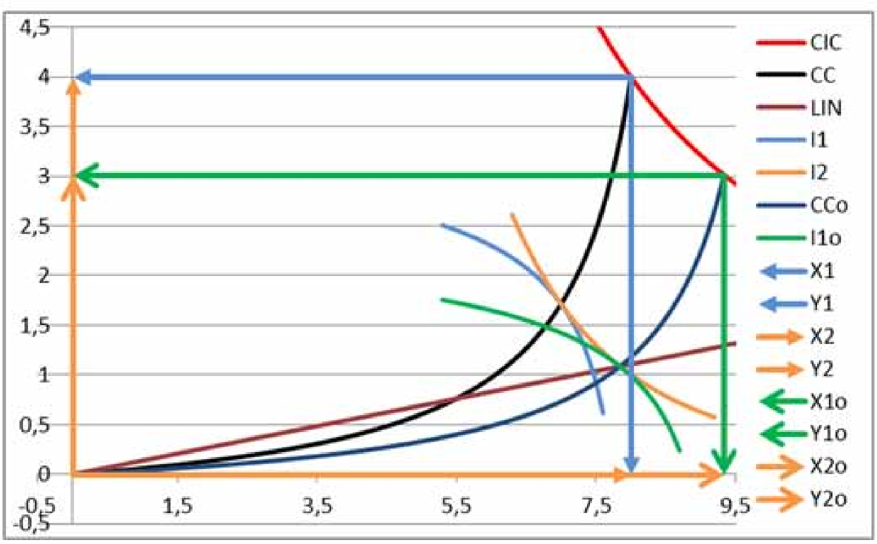
Having described the necessary parameters, the optimal size of resources for fixed income can be determined.

For given fixed incomes $\overline{I_1}$ and $\overline{I_2}$ (9), (10), we find the values of resources L^0 and K^0 with CIC for which incomes $\overline{I_1^0}$ and $\overline{I_2^0}$ will be optimal in the sense of the statement of problem (2) (Fig. 10).

Using formulas (35), (36) $L^0 \, and \, K^0$ can be obtained:

$$L^0 = \frac{\left(\delta_{12}\right)^{1-\alpha_1}}{p_1\beta_1\gamma_{12}^{1-\alpha_1}}\overline{I_1^0} + \frac{\left(\delta_{12}\right)^{1-\alpha_2}}{p_2\beta_2}\overline{I_2^0} \tag{38}$$

Figure 10. Graphical determination of the value of resources L^0 and K^0, for which fixed incomes $\overline{I_1}$ and $\overline{I_2}$ will be optimal $\overline{I_1^0}$ and $\overline{I_2^0}$ according to formulas (5), (33), (38), (39)



$$K^0 = \frac{\gamma_{12}^{\alpha_1}}{p_1\beta_1\left(\delta_{12}\right)^{\alpha_1}}\overline{I_1^0} + \frac{1}{p_2\beta_2\left(\delta_{12}\right)^{\alpha_2}}\overline{I_2^0} \tag{39}$$

As a result the optimal size of resources at a fixed total income can be determined.

For a given fixed optimal income $\overline{I^0}$ (in the sense of the statement of problem (2)), we find the values of resources *L* and *K*, at which it will not change (Fig. 11). From formulas (37) and (1) we obtain:

$$K = \begin{cases} \dfrac{\left(\dfrac{\overline{I^0}}{p_2\beta_2}\right)^{\frac{1}{1-\alpha_2}}}{\dfrac{\alpha_2}{L^{1} - \alpha_{2}}}, L \le \dfrac{\alpha_1}{\alpha_2}\dfrac{\overline{I^0}}{p_2\beta_2}\left(\delta_{12}\right)^{1-\alpha_2} & (a) \\ \dfrac{\bar{I^0}}{p_2\beta_2(1-\alpha_2)(\delta_{12})^{\alpha_2}} - \dfrac{\alpha_2}{(1-\alpha_2)\delta_{12}}L, \dfrac{\alpha_1}{\alpha_2}\dfrac{\bar{I^0}}{p_2\beta_2}(\delta_{12})^{1-\alpha_2} < L < \dfrac{\bar{I^0}}{p_2\beta_2}\left(\delta_{12}\right)^{1-\alpha_2} & (b) \\ \dfrac{\left(\dfrac{\overline{I^0}}{p_1\beta_1}\right)^{\frac{1}{1-\alpha_1}}}{\dfrac{a_1}{L^{1-\alpha_1}}}, L \ge \dfrac{\overline{I^0}}{p_2\beta_2}(\delta_{12})^{1-\alpha_2} & (c) \end{cases} \tag{40}$$

The graph of the expression (39) will be the set of points (*L, K*) (resource constraints) at which the specified optimal income $\overline{I^0}$ does not change. Therefore, curve of expression (40) can be called the **community income indifference curve** (CIIC) (Fig. 11).

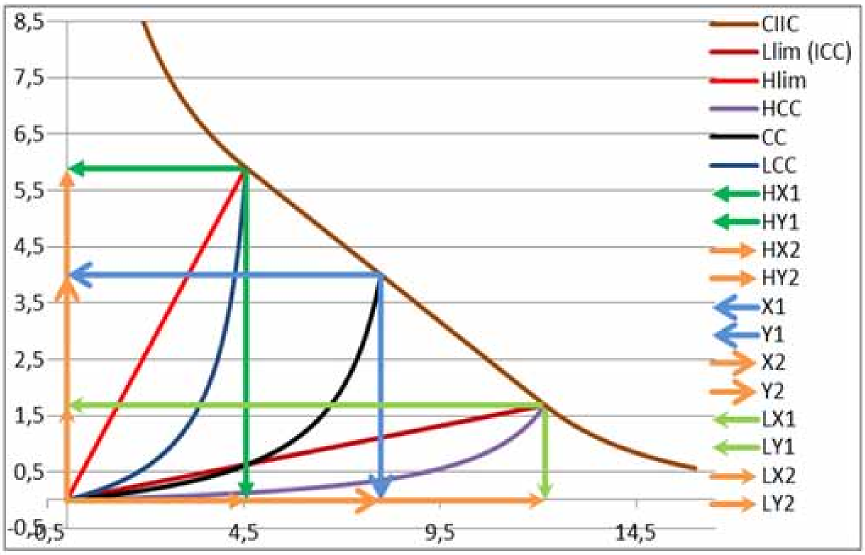
To realize a given total income $\overline{I^0}$ and at the same time produce both types of products, it is necessary that the constraints on resources (*L, K*) are on the CIIC given by (40b). This part of CIIC will be located between two straight lines - lower Llim and upper Hlim boundaries (Fig. 11) (fulfilment of condition (34)) by the given equations:

$$Llim = \frac{1}{\delta_{12}}X_2 \tag{41}$$

$$Hlim = \frac{\gamma_{12}}{\delta_{12}}X_2 \tag{42}$$

With an agreement on the implementation of any state $\left(X_2, \frac{1}{\delta_{12}}X_2\right)$ from the lower boundary Llim (41), it is possible to determine the resource boundaries (*L, K*) with CIIC (40b) at which the total income $\overline{I^0}$ will not change. Therefore, the lower bound Llim (41) can be called the income contract curve (ICC) (Fig. 11).

Figure 11. Graphical determination of community income indifference curve (CIIC) and income contract curve (ICC) by the formulas (5) and (33), (40)



4. CONCLUSION

This study analytically unified the contract curve, community indifference curve, production possibility frontier under the assumption of the homogeneous production functions. Theis is achieved under the Cobb-Douglas production function to solve the resource allocation problem aimed at revenue maximization. Such theoretical concepts as income contract curve and community income indifference curve are discussed. The obtained results can be further used as a tool for solving theoretical and practical problems in this area. By these means the current study provides a substantial contribution to the development of game theory, in particular – a two parties bargaining game.

The present study is limited in that it considers a fixed number (i.e., two) of production functions. Therefore, further research may seek to extend the results obtained to a more general case with three or more production functions. Also, the current study is limited to homogeneous Cobb-Douglas production functions. Another direction of future research could be aimed at solving income optimization problems under the Cobb-Douglas production functions with increasing and decreasing returns to scale.

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